Conformal Cosmology and the Age of the Universe*

Philip D. Mannheim
Department of Physics, University of Connecticut, Storrs, CT 06269
mannheim@uconnvm.uconn.edu

Abstract

We show that within the cosmology associated with conformal gravity the age of the universe is given as $1/H_0$, to thus overcome the current cosmological age crisis. We show that while the parameter $\Omega_{mat} = \rho_{mat}/\rho_C$ takes on all values between zero and infinity in conformal gravity, nonetheless it is of order one (but not identically equal to one) for half a Hubble time to thus naturally explain its current closeness to one without fine tuning. We show that the cosmological constant is naturally of order the energy density ρ_{mat} of ordinary matter again without fine tuning. We compare and contrast conformal cosmology with that of the standard Friedmann cosmology.

For a theory which has long since been declared to be the true and correct cosmological theory, the standard Friedmann model is currently beset by a surprisingly large number of problems. The definitive new Hubble Space Telescope (Freedman et al 1994) determination of the value of the Hubble parameter H_0 causes the standard $\Omega_{mat} = 1$, $\Omega_{vac} = 0$ Friedmann model age $t_0 = 2/3H_0$ for the universe to now be less than that of some of its constituents. While the age prediction for the model can be increased by allowing for a non-zero cosmological constant vacuum energy contribution Ω_{vac} (see e.g. Krauss and Turner 1995 who actually argue for such a non-zero value from a variety of cosmological considerations), unless this contribution is constrained according to the inflationary universe requirement $\Omega_{tot} = \Omega_{mat} + \Omega_{vac} = 1$, the celebrated flatness problem would then reappear. However, the fine tuning problem which would then be required of Ω_{vac} to enforce this desired $\Omega_{tot} = 1$ would then be no less than 60 or so orders of magnitude more severe than the flatness problem tuning problem for which inflation was proposed in the first place (Guth 1981). New

*UCONN 95-08, December 1995

cosmological data are thus forcing cosmologists to finally have to confront the one problem that they had previously side stepped by setting $\Omega_{vac} = 0$ (not that any reason had been advanced for that choice either). Beyond these already quite severe issues, new abundance determinations and computational analyses are calling into question (White et al 1993, Hata et al 1995, Copi, Schramm and Turner 1995) what had always been regarded as the primary achievement of the standard model, namely big bang nucleosynthesis.

While it is of course much too early to draw any definitive conclusions regarding the ultimate status of the standard model, nonetheless the current situation does demand a critical reappraisal of its basic ingredients, with the most basic of all being its reliance on the use of Newton-Einstein gravity in the first place, an issue which Mannheim and Kazanas have actually been challenging in a recent series of papers simply by noting that there is currently no known principle which would uniquely select out the Einstein-Hilbert action from amongst the infinite class of all order covariant metric based theories of gravity that one could in principle at least consider. Motivated by the fact (Mannheim 1990) that the assumption of an underlying conformal symmetry (viz. invariance under local conformal transformations of the form $g_{\mu\nu}(x) \to \Omega(x) g_{\mu\nu}(x)$ and the consequently unique conformal invariant gravitational action $I_W = -\alpha \int d^4x(-g)^{1/2} C_{\lambda\mu\nu\kappa} C^{\lambda\mu\nu\kappa}$) actually strictly forbids the presence of any fundamental cosmological constant (to thereby provide a symmetry based framework with which to address this longstanding problem), Mannheim and Kazanas then embarked on a detailed analysis of the possible astrophysical implications of the conformal gravity theory. They solved for the exact exterior (Mannheim and Kazanas 1989; see also Riegert 1984) and interior (Mannheim and Kazanas 1994) metrics associated with a static, spherically symmetric source in the model, demonstrated their consistency, and found that in conformal gravity all the classic tests of General Relativity could still be met (even despite the absence of the Einstein-Hilbert action which is also forbidden by the conformal invariance - to incidentally thereby demote the Planck length L_{Pl} from fundamental status and decouple it from quantum gravity fluctuations). Further, it was shown that the theory actually

departs from the standard theory on galactic distance scales in a manner (Mannheim 1993a, 1995) which can provide for a resolution of the galactic rotation curve problem without the need to introduce any dark matter, this dark matter of course being the primary and still totally elusive component of the standard $\Omega_{mat} = 1$ paradigm. Moreover, a first conformal cosmological model was constructed (Mannheim 1992) and it was shown to naturally resolve the flatness problem by necessarily possessing a $k \ll 0$ and thus far from flat topology. Subsequently (Mannheim 1995) it was found that this very negative curvature acts universally on the galaxies which make up the Hubble flow to completely explain the departures of their rotation curves from the luminous Newton expectation without any need for dark matter, while also automatically enforcing the universal Tully-Fisher relation. Moreover, an actual value for k, viz. $k = -3.5 \times 10^{-60}$ cm^{-2}, was even extracted from the rotation curve data. In the present paper we continue the study of Mannheim (1992) to calculate the age of the Universe in the model and to show that the current value of Ω_{mat} is naturally of order one.

While we have already noted that the very fact of conformal invariance forces us to change the gravitational side of the gravitational equations of motion, with the Bach tensor (the variation of the conformal action I_W with respect to the metric) then replacing the familiar Einstein tensor, in a sense a possibly even more far reaching aspect of conformal invariance is that it also forces us to change the structure of the matter energy-momentum tensor side as well, thereby forcing us to reconsider (Mannheim 1993b, Mannheim and Kazanas 1994) the prevailing Newtonian 'billiard ball' perfect fluid view of gravitational sources familiar in the standard applications of gravity to astrophysical situations. Specifically, since conformal symmetry would require strictly massless matter fields, it is necessary to introduce scalar (Higgs) fields whose non-vanishing vacuum expectation values would then spontaneously break the conformal symmetry to give masses to the matter fields. Such scalar fields would then also carry energy and momentum which also couple to gravity (this energy and momentum is simply ignored in the standard 'billiard ball' model of sources, even in fact when the Higgs fields are taken to be the conventional Weinberg-Salam fields which are currently

thought to generate particle masses); and as we shall see, because of the underlying conformal structure, the contributions of these scalar fields to the energy-momentum tensor prove to not only be too significant to be ignored, but they also turn out to be constrained in a way which enables us to address many current astrophysical puzzles.

To see what the constraints of conformal symmetry explicitly entail, consider the typical case of fermionic matter fields Yukawa coupled to scale breaking scalars. For them, the most general covariant, conformal invariant matter action I_M takes the form

$$I_M = -\int d^4x(-g)^{1/2}[\hbar S^\mu S_\mu/2 + \lambda S^4 - \hbar S^2 R^\mu{}_\mu/12 + i\hbar\bar{\psi}\gamma^\mu(x)(\partial_\mu + \Gamma_\mu(x))\psi - hS\bar{\psi}\psi] \quad (1)$$

where $\Gamma_\mu(x)$ is the fermion spin connection and h and λ are dimensionless coupling constants. (For simplicity we only consider fermion bilinears in I_M. In a more detailed model we would also need to consider fields which transform as fermion quadrilinears as well with those fields being responsible for scale breaking in the massless, high temperature, cosmological radiation era.) When the scalar field $S(x)$ in I_M acquires a non-vanishing vacuum expectation value (which we are free to set equal to a spacetime independent constant S because of the conformal invariance), the fermion then obeys the Dirac equation $i\hbar\gamma^\mu(x)[\partial_\mu + \Gamma_\mu(x)]\psi = hS\psi$ and acquires a mass hS. Once S is constant, we note that this Dirac equation is the same as the one which is used for fermions with mechanical masses (viz. 'billiard balls'), so that the geodesic motion for massive fermions which follows from it is the standard one. For macroscopic purposes we note that the incoherent averaging of the fermion kinetic energy operator $i\hbar\bar{\psi}\gamma^\mu(x)(\partial_\mu + \Gamma_\mu(x))\psi$ over all the occupied positive frequency modes of this Dirac equation leads us (Mannheim 1992) to a standard kinematic perfect fluid of these fermions with energy-momentum tensor $T^{\mu\nu}_{kin} = (\rho_{mat} + p_{mat})U^\mu U^\nu + p_{mat}g^{\mu\nu}$; while the averaging of the total energy-momentum tensor and of the scalar field equation of motion associated with Eq. (1) lead us to

$$T^{\mu\nu} = (\rho_{mat} + p_{mat})U^\mu U^\nu + p_{mat}g^{\mu\nu} - \hbar S^2(R^{\mu\nu} - g^{\mu\nu}R^\alpha{}_\alpha/2)/6 - g^{\mu\nu}\lambda S^4$$
$$= (\rho_{mat} + p_{mat})U^\mu U^\nu + p_{mat}g^{\mu\nu} - g^{\mu\nu}(3p_{mat} - \rho_{mat})/4 - \hbar S^2(R^{\mu\nu} - g^{\mu\nu}R^\alpha{}_\alpha/4)/6 \quad (2)$$

and

$$\hbar S^2 R^{\mu}{}_{\mu} - 24\lambda S^4 + 6(3p_{mat} - \rho_{mat}) = 0 \tag{3}$$

respectively. It is important to note that the $-g^{\mu\nu}(3p_{mat} - \rho_{mat})/4$ term displayed in the second form of Eq. (2) arises from the incoherent averaging of the Yukawa $-g^{\mu\nu}\hbar S\bar{\psi}\psi/4$ term and is needed to maintain the tracelessness of the full conformal $T^{\mu\nu}$, ($T^{\mu\nu}_{kin}$ itself of course is not traceless). Since the total $T^{\mu\nu}$ is also covariantly conserved, we see from the first form of Eq. (2) that $T^{\mu\nu}_{kin}$ is conserved all on its own, with the sum of all the other terms in the total $T^{\mu\nu}$ being independently conserved also. Thus all the standard features that arise from the covariant conservation of $T^{\mu\nu}_{kin}$ (such as the dependence of the cosmological ρ_{mat} on $R(t)$) continue to hold in the conformal theory, with the motions of the matter fields being exactly the same as they would have been had the matter fields in fact been billiard balls. However, since gravity couples to the entire $T^{\mu\nu}$ and not merely to $T^{\mu\nu}_{kin}$, its behavior is radically affected by the presence of all these additional non-Newtonian terms in the full $T^{\mu\nu}$, and so we now explore their implications for cosmology.

For applications of conformal gravity to cosmology we note that in geometries such as Robertson-Walker which are conformal to flat the conformal Bach tensor $(\delta I_W/\delta g_{\mu\nu})$ vanishes identically, so that the matter fields are constrained to obey the equation of motion $T^{\mu\nu} = 0$. Given Eqs. (2) and (3), we thus see that since $T^{\mu\nu} = 0$ in conformal cosmology, the terms in $T^{\mu\nu}$ that depend on S (which collectively constitute a general cosmological term which includes both a cosmological constant and a back reaction on the geometry) must between them add up to the energy density in ordinary matter, i.e. the magnitude of the macroscopic S is fixed by how many fermion states are occupied in ρ_{mat} in the first place. Thus we find that not only does the conformal theory possess no fundamental cosmological term, the one which is subsequently induced by the symmetry breaking scalar field adjusts itself self-consistently via the back reaction of the scalar field on the geometry to acquire a scale which is fixed by the energy density of the matter which got its mass from the selfsame scalar field, so that the full cosmological term is thus neither smaller nor larger in magnitude than

the energy density of ordinary matter, to thus naturally fix the positive frequency mode contribution to Ω_{vac} without fine tuning. Conformal cosmology thus naturally addresses the issue of the magnitude of the cosmological term by using its symmetry constraints. This situation should be contrasted with that of the standard model, a model which has no such constraints, and in which the self-consistent back reaction on the geometry is not even considered - in fact the standard model cosmological term is identified as $g^{\mu\nu}V_{min}(S)$ where the minimum value $V_{min}(S)$ of the potential is simply transported from flat space without regard to any of the other terms present in Eq. (2) or to their mutual self-consistency. As we now see, the cosmological constant problem should not in fact be viewed as a generic problem for cosmology, but rather as a specific feature of the Einstein Equations, with the issue for the standard theory being how to get rid of a term which has no reason not to be there.

As regards the value of Ω_{mat} in the model, we note that in a Robertson-Walker geometry the condition $T^{\mu\nu} = 0$ reduces to

$$\dot{R}^2(t) + \frac{2R^2(t)c\rho_{mat}}{\hbar S^2} = -kc^2 - \frac{2R^2(t)\lambda S^2 c^2}{\hbar} \tag{4}$$

to yield a condition which only differs from the analogous standard model equation in one regard, namely that the quantity $-\hbar S^2/12$ has replaced $c^3/16\pi G$ in the second term on the left hand side. From the point of view of the standard model, Eq. (4) would have been obtained in standard gravity if standard gravity were repulsive rather than attractive, with the back reaction of the scalar field on the geometry in conformal gravity thus acting like an induced effective repulsive rather than attractive gravitational term in the conformal case. Because of this crucial change in sign, the $\dot{R}^2$ and $2R^2c\rho_{mat}/\hbar S^2$ terms are required to add in Eq. (4) rather than cancel so that the fine tuning flatness problem present in the standard model (where these two huge quantities have to cancel to an extraordinary degree of accuracy) is thus not encountered in the conformal case (Mannheim 1992). Moreover, Eq. (4) can only have solutions at all if $k \ll 0$ (unless the coefficient λ is overwhelmingly

negative, with it in fact generally even being believed to be positive), and thus leads us to an automatically open and very negatively curved Universe. (Essentially the only way the geometry can cancel the positive energy density of ordinary matter and maintain $T^{\mu\nu} = 0$ is if the gravitational field itself contains the negative energy associated with negative spatial curvature.) Now while we have shown that the conformal model naturally avoids the flatness problem, nonetheless Ω_{mat} is still quite close to one today, a fact which also must be natural in our model, and it is to this issue which we now turn.

It is most straightforward to discuss the general implications of Eq. (4) for Ω_{mat} in the simplified situation in which λ is set equal to zero. Since the matter era does not appear to possess solutions which can readily be expressed in terms of elementary functions, it is simpler to consider the radiation era. On setting $\rho_{mat} = A/R^4 = \sigma T^4$, Eq. (4) is readily integrated to yield

$$R^2(t) = -kc^2t^2 + R^2_{min} \quad , \quad H(t) = \frac{1}{t(1 - R^2_{min}/kc^2t^2)} \quad , \quad q(t) = \frac{R^2_{min}}{kc^2t^2} \tag{5}$$

so that $R(t)$ has a finite minimum radius $R_{min} = (-2A/\hbar k S^2 c)^{1/2}$ at $t = 0$ (and thus a finite maximum temperature T_{max}) with the cosmology thus being singularity free (precisely because it induces a repulsive gravitational component so that conformal gravity can protect itself from its own singularities - something of course not the case in standard gravity). In terms of the conventionally defined and very convenient quantity $\Omega_{mat}(t) = \rho_{mat}/\rho_C$, the temperature at time t then obeys

$$\frac{T^2_{max}}{T^2(t)} = 1 - \frac{kc^2t^2}{R^2_{min}} = 1 - \frac{1}{q(t)} = 1 + \frac{4\pi L^2_{Pl} S^2}{3\Omega_{mat}(t)} \tag{6}$$

Thus for $\Omega_{mat}(t_0)$ currently of order one the scale parameter S must be at least as big as $10^{10} L^{-1}_{Pl}$ if the early Universe is to have a maximum temperature of at least 10^{10} degrees. Analogously, the current value of the deceleration parameter must obey $q(t_0) \leq 10^{-20}$. (To get larger phenomenological values for $q(t_0)$ would require the reintroduction of the λ term of Eq. (4).) Then since according to Eq. (5) the Hubble and deceleration parameters are related as $H(t)(1 - q(t)) = 1/t$ in the model, it follows that the age of the Universe is given

as $t_0 = 1/H(t_0) = 1/H_0$, to be compared with $t_0 = 1/2H_0$ in the standard model radiation era. In fact, as we show below, even in the matter era the age remains $1/H_0$ in the conformal theory, and thus yields an age which is currently phenomenologically viable.

Now according to Eq. (6), $\Omega_{mat}(t)$ goes through all values from infinity to zero during the lifetime of the Universe, and thus must pass through one at some time, and as we have just shown there even exists a value of S for which $\Omega_{mat}(t_0)$ is of order one today (though not identically equal to one). Nonetheless, we still need to ask whether we are likely to be at that value today since $\Omega_{mat}(t)$ could possibly be close to one only for a very short time, to then require some fine tuning to get it close to one in the current epoch. To resolve this issue we note from Eqs. (5) and (6) that at time $t = t_0/2$ we obtain

$$\frac{T^2(t_0/2)}{T^2(t_0)} = \frac{-kc^2t_0^2 + R_{min}^2}{-kc^2t_0^2/4 + R_{min}^2} = \frac{1 - q(t_0)}{1/4 - q(t_0)} = 4 \quad , \quad \frac{\Omega_{mat}(t_0/2)}{\Omega_{mat}(t_0)} = \frac{t_0^2}{t_0^2/4} = 4 \tag{7}$$

so that both $T(t)$ and $\Omega_{mat}(t)$ take values close to their current values for no less than half a Hubble time. Thus even though their early Universe values differ radically from their current values, the probability of finding $T(t)$ and $\Omega_{mat}(t)$ in their current values at the current time is still very high. In this way the model explains why $\Omega_{mat}(t_0)$ can naturally be of order one today despite its radically different values at very early times. In contrast, we recall that the flatness problem for the standard model stems from the fact that given the closeness of $\Omega_{mat}(t_0)$ to one today, the Friedmann evolution equations require $\Omega_{mat}(t)$ to be even closer to one at earlier times. Thus we see that the flatness problem is not in fact generic to cosmology, but rather it would appear to be a specific feature of the Einstein Equations, and may thus even be a signal that the Einstein Equations might not be the appropriate ones for cosmology.

It is also of interest to see how the evolution equation of Eq. (4) itself manages to avoid any fine tuning problem. In the solution of Eq. (5) the two terms on the left hand side of Eq. (4) respectively take the form:

$$\dot{R}^2(t) = \frac{k^2c^4t^2}{-kc^2t^2 + R_{min}^2} = \frac{-kc^2}{(1 - q(t))} \quad , \quad \frac{2Ac}{\hbar S^2 R^2(t)} = \frac{-kc^2 R_{min}^2}{-kc^2t^2 + R_{min}^2} = \frac{kc^2 q(t)}{(1 - q(t))} \tag{8}$$

to thus give radically different time behaviors to these two terms even while their sum remains constant ($=-c^2k$). Specifically, $\dot{R}^2$ begins at zero and slowly goes to $-c^2k$ at late times, while $2Ac/\hbar S^2R^2$ does the reverse as it goes to zero from an initial value of $-c^2k$. Moreover, evaluating them today then shows that both the terms have already attained their late values, and that rather than being of the same order of magnitude today, in fact the $\dot{R}^2(t_0)$ term is 10^{20} orders of magnitude larger than the $2Ac/\hbar S^2R^2(t_0)$ term. This behavior differs radically from that found in the standard model (where the analogous two terms are both of the same order of magnitude today) simply because the scale factor S of the conformal model is not of order L_{Pl}^{-1} but rather a factor at least 10^{10} times bigger. It is also of interest to ask at what time the two terms given in Eq. (8) were in fact of the same magnitude. From Eq. (8) we see that this would occur when $q(t) = -1$, i.e. at a time $t = t_0/10^{10}$, a time at which $T(t) = T_{max}/\sqrt{2}$ which is well in the early Universe; and in passing we note that in the conformal case the Universe initially cools very slowly dropping in temperature by a factor of only $\sqrt{2}$ in its first 10^7 sec.

It is also possible to extend the age estimate for the Universe to the matter era where $\rho_{mat} = B/R^3$, a relation which fixes the magnitude of S anew in accord with Eq. (3). In this era the Universe is found to have a minimum radius given by $R_{min} = -2B/kS^2\hbar c$ (we again set $\lambda = 0$ for simplicity), and an evolution given by

$$(-k)^{1/2}ct = R_{min}\, log\left(\frac{R^{1/2} + (R - R_{min})^{1/2}}{R_{min}^{1/2}}\right) + R^{1/2}(R - R_{min})^{1/2} \qquad (9)$$

and

$$\frac{R(t)}{R_{min}} = \frac{T_{max}}{T(t)} = 1 + \frac{4\pi L_{Pl}^2 S^2}{3\Omega_{mat}(t)} = \frac{-kc^2}{-kc^2 - H^2(t)R^2(t)} \qquad (10)$$

Thus again we find that $S \gg L_{Pl}^{-1}$ if T_{max} is to be very big, with $R_0^2H_0^2$ then being extremely close to $-kc^2$ today. However, since R_0 is very much greater than R_{min}, it follows from Eq. (9) that $-kc^2t_0^2 = R_0^2$ today, so that the age of the Universe is again given as $t_0 = 1/H_0$ as required. Since the standard model matter era yields an age $t_0 = 2/3H_0$, we see that conformal gravity yields an age which is 50% bigger, so that its age prediction for a Hubble

parameter $h = 0.75$ (defining $H_0 = 100h$ km sec^{-1} Mpc^{-1}) is the same as that of a standard model with the now excluded value of $h = 0.5$; with the new HST value (Freedman et al 1994) of $h = 0.80 \pm 0.17$ actually yielding an age $1/H_0 = 12.2 \pm 2.6$ Gyr which is compatible with the globular cluster age estimate of 16 ± 3 Gyr quoted by Krauss and Turner (1995).[1]

As we thus see, in both the radiation and matter eras the age of the Universe is given as $t_0 = 1/H_0$, a result that could have been read off directly from Eq. (4) in the limit in which we drop the energy density ρ_{mat} altogether, viz. the pure curvature dominated limit in which $\dot{R}^2(t) = -kc^2$ and $R(t) = (-k)^{1/2}ct$ (this also being a horizon free limit in which the particle horizon $d(t) = R(t)\int_0^t dt/R(t)$ is infinite).[2] In fact this curvature dominance drives the cosmology, a fact that could have been anticipated from Eq. (4), with this curvature dominance causing the Universe to expand far more slowly than in the standard case. While this curvature dominated cosmology is thus seen to be able to address some outstanding puzzles of the standard model, it is not itself yet completely free of problems, since this same slow expansion seems to be able to only produce substantial amounts of primordial helium and appears to have trouble generating other light elements (Knox and Kosowsky 1993; Elizondo and Yepes, 1994). Whether this is simply a property of using just the simple cosmology based on Eq. (1) and/or whether it could be resolved

[1]While it is completely standard to compare the age of the Universe with that of its constituents in the above way, it is perhaps worth noting in passing that this procedure is only an approximate one, and that its level of accuracy is only ascertainable by actually making a general coordinate transformation between the time coordinate of the comoving Robertson-Walker frame associated with the expansion of the Universe and that of each Schwarzschild coordinate rest frame system in which the age of each constituent is measured.

[2]Moreover, in the presence of matter straightforward calculation shows that for the form of $R(t)$ given in Eq. (5), the (dimensionless) ratio of the horizon size to the spatial radius of curvature $R_{curv}(t)$ $(= (6/R^{(3)})^{1/2}$ where $R^{(3)}$ is the modulus of the Ricci scalar of the spatial part of the metric) is given as $d(t)/R_{curv}(t) = (-k)^{1/2}cd(t)/R(t) = log[T_{max}/T + (T_{max}^2/T^2 - 1)^{1/2}]$ where $T_{max} = (4\pi/3\Omega_{mat}(t_0))^{1/2}T(t_0)SL_{Pl}$ according to Eq. (6). This ratio is thus much greater than one at recombination to thus naturally solve the horizon problem in the conformal model. For comparison with the standard theory, we recall that when its spatial curvature is negative, the standard theory yields $d(t)/R_{curv}(t) = log[T_{ref}/T + (T_{ref}^2/T^2 + 1)^{1/2}]$ $(T_{ref} = (1/\Omega_{mat}(t_0) - 1)^{1/2}T(t_0))$ a ratio which is much smaller than one at recombination. Comparing the two expressions for the ratio we see that they only differ substantially at recombination because the conformal inverse length scale S is much greater than L_{Pl}^{-1}, a feature which also enabled us to resolve the flatness problem as we discussed above. As we thus now see, the origin of both the flatness and horizon problems in the standard theory stems from the fact that cosmological observables are apparently not naturally parameterized in terms of the inverse length scale associated with Newton's constant, but rather by one which is orders of magnitude bigger.

in more detailed dynamical conformal models remains to be addressed. However, since the standard cosmology is also having nucleosynthesis problems (and the standard cosmology has yet, despite the prevailing view on dynamical generation of particle masses, to explain exactly why it models the entire self-consistent mass generating $T^{\mu\nu}$ of Eq. (2) purely by its mechanical 'billiard ball' kinematic perfect fluid $T^{\mu\nu}_{kin}$ piece),[3] more detailed study of this issue might prove fruitful; and since the conformal theory does seem to be able to nicely address so many other outstanding cosmological puzzles in such a straightforward manner it would appear to merit further study.

The author would like to thank M. Turner for asking the right question. This work has been supported in part by the Department of Energy under grant No. DE-FG02-92ER40716.00.

References

[1] Copi, C. J., Schramm, D. N., and Turner, M. S. 1995, Phys. Rev. Lett. 75, 3981.

[2] Elizondo, D., and Yepes, G. 1994, ApJ, 428, 17.

[3] To understand the exact nature of this assumption, we note that from purely geometric considerations alone the most general energy-momentum tensor in a Robertson-Walker geometry must take the generic form $T^{\mu\nu} = (A(t) + B(t))U^\mu U^\nu + B(t)g^{\mu\nu}$, with the coefficients $A(t)$ and $B(t)$ in general not necessarily being obliged to be proportional to each other. Relations between $A(t)$ and $B(t)$ only follow from specific dynamical assumptions. Moreover, if $A(t)$ and $B(t)$ contain two or more separate components (this being the case for the conformal $T^{\mu\nu}$ of Eq. (2)), then even if the separate components are related via $A_1 = w_1 B_1$, $A_2 = w_2 B_2$, it does not follow that $A_1 + A_2$ is proportional to $B_1 + B_2$. Further, even for the restricted case of the kinematic $T^{\mu\nu}_{kin}$ where $A(t)$ and $B(t)$ are in fact associated with a standard perfect fluid, it turns out that they are still not in fact proportional to each other at all temperatures. Specifically, consider an ideal N particle classical gas of particles of mass m in a volume V at a temperature T. For this system the Helmholtz free energy $A(V,T)$ is given as $\exp[-A(V,T)/NkT] = V\int d^3p\, \exp[-(p^2+m^2)^{1/2}/kT]$, so that the pressure takes the simple form $P = -(\partial A/\partial V)_T = NkT/V$, while the internal energy $U = A - T(\partial A/\partial T)_V$ evaluates in terms of Bessel functions as $U = 3NkT + NmK_1(m/kT)/K_2(m/kT)$. In the two limits $m/kT \to 0$, $m/kT \to \infty$ we then find that $U \to 3NkT$, $U \to Nm + 3NkT/2$. Thus only at these two extreme temperature limits does it follow that the energy density and the pressure are in fact proportional, with their relation in intermediate regimes such as the transition region from the radiation to the matter era being far more complicated. Since we thus see that the generic $A(t)$ and $B(t)$ are not in fact proportional to each other in general and at all temperatures, it would be interesting to see to what degree the standard model nucleosynthesis predictions are sensitive to more general choices for $A(t)$ and $B(t)$ than conventionally considered.

[3] Freedman, W. L., Madore, B. F., Mould, J. R., Hill, R., Ferrarese, L., Kennicutt Jr., R. C., Saha, A., Stetson, P. B., Graham, J. A., Ford, H., Hoessel, J. G., Huchra, J., Hughes, S. M., and Illingworth, G. D 1994, Nature, 371, 757.

[4] Guth, A. H. 1981, Phys. Rev. D23, 347.

[5] Hata, N., Scherrer, R. J., Steigman, G., Thomas, D., Walker, T. P., Bludman, S., and Langacker, P. 1995, Phys. Rev. Lett. 75, 3977.

[6] Knox, L., and Kosowsky, A. 1993, "Primordial Nucleosynthesis in Conformal Weyl Gravity", Preprint Fermilab-Pub-93/322-A, November, 1993.

[7] Krauss, L. M., and Turner, M. S. 1995, "The Cosmological Constant is Back", Preprint CWRU-P6-95, April, 1995.

[8] Mannheim, P. D. 1990, GRG, 22, 289.

[9] Mannheim, P. D. 1992, ApJ, 391, 429.

[10] Mannheim, P. D. 1993a, ApJ, 419, 150.

[11] Mannheim, P. D. 1993b, GRG, 25, 697.

[12] Mannheim, P. D. 1995, "Cosmology and Galactic Rotation Curves", Preprint UCONN 95-07, November, 1995.

[13] Mannheim, P. D., and Kazanas, D. 1989, ApJ, 342, 635.

[14] Mannheim, P. D., and Kazanas, D. 1994, GRG, 26, 337.

[15] Riegert, R. J. 1984, Phys. Rev. Lett. 53, 315.

[16] White, S. D. M., Navarro, J. F., Evrard, A. E., and Frenk, C. S. 1993, Nature, 366, 429.